PROPOSTAS DE RESOLUÇÃO

Capítulo 7

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1. $f(x)=\frac{1}{\sqrt{x}}$

$F(x)=2\sqrt{x}+C$

Como $F(4)=4$ então $2\sqrt{x}+C=4\Leftrightarrow C=0$ logo a primitiva pedida é:

$F(x)=2\sqrt{x}$.

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2.1 $\int(-7)\,dx=-7x+C$

2.2 $\int\left(-\frac{2}{3}\right)dx=-\frac{2}{3}x+C$

2.3 $\int\sqrt{2}dx=\sqrt{2}x+C$

2.4 $\int(-7x)\,dx=-7\int xdx=-7\frac{x^2}{2}+C=-\frac{7}{2}x^2+C$

2.5 $\int\left(\frac{7}{5}x\right)dx=\frac{7}{5}\int xdx=\frac{7}{5}\frac{x^2}{2}+C=\frac{7}{10}x^2+C$

2.6 $\int x^3dx=\frac{x^{3+1}}{3+1}+C=\frac{x^4}{4}+C$

2.7 $\int x^{80}dx=\frac{x^{80+1}}{80+1}+C=\frac{x^{81}}{81}+C$

2.8 $\int\sqrt[3]{x}dx=\int x^{\frac{1}{3}}Cx=\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C=$

$=\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+C=\frac{3}{4}\sqrt[3]{x^4}+C$

2.9 $\int x^{-10}dx=\frac{x^{-10+1}}{-10+1}+C=\frac{x^{-9}}{-9}+C=-\frac{1}{9x^9}+C$

2.10 $\int\left(x^5+x^4\right)dx=\int x^5dx+\int x^4dx=$

$=\frac{x^{5+1}}{5+1}+\frac{x^{4+1}}{4+1}+C=\frac{x^6}{6}+\frac{x^5}{5}+C$

2.11 $\int\left(8x^2-7x+\frac{1}{2}\right)dx=$

$=8\int x^2dx-7\int xdx+\frac{1}{2}\int dx=$

$=8\frac{x^3}{3}-7\frac{x^2}{2}+\frac{1}{2}x+C=$

$=\frac{8}{3}x^3-\frac{7}{2}x^2+\frac{1}{2}x+C$

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3.1 $\int x^{10}dx=\frac{x^{10+1}}{10+1}+C=\frac{x^{11}}{11}+C$

3.2 $\int\frac{1}{\sqrt{x^3}}dx=\int x^{-\frac{3}{2}}dx=\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}+C=$

$=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C=-\frac{2}{\sqrt{x}}+C$

3.3 $\int\frac{5}{\sqrt{5x^3}}dx=\frac{5}{\sqrt{5}}\int x^{-\frac{3}{2}}dx=\sqrt{5}\left(-\frac{2}{\sqrt{x}}\right)+C=$

$=-\frac{2\sqrt{5}}{\sqrt{x}}+C$

3.4 $\int\left(e^x+3^x-\sin x+3\right)dx=$

$=\int e^x\,dx+\int 3^x\,dx-\int\sin xdx+3\int dx=$

$=e^x+\frac{3^x}{\ln 3}+\cos x+3x+C$

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4.1 $\int x\ln xdx$

Seja $u=\ln x\Rightarrow du=\frac{1}{x}dx$

$dv=xdv\Rightarrow v=\frac{x^2}{2}$

$\int x\ln xdx=\frac{x^2}{2}\ln x-\int\left(\frac{x^2}{2}\times\frac{1}{x}\right)dx=$

$=\frac{x^2}{2}\ln x-\frac{1}{2}\int xdx=$

$=\frac{x^2}{2}\ln x-\frac{1}{2}\frac{x^2}{2}+C=$

$=\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$

4.2 $\int x\,e^{2x}\,dx$

Seja $u=x\Rightarrow du=dx$

$dv=e^{2x}\,dv\Rightarrow v=\frac{1}{2}e^{2x}$

$\int x\,e^{2x}\,dx=\frac{1}{2}x\,e^{2x}-\int\frac{1}{2}e^{2x}\,dx=$

$=\frac{1}{2}x\,e^{2x}-\frac{1}{2}\int e^{2x}\,dx=$

$=\frac{1}{2}x^{2x}-\frac{1}{2}\left(\frac{1}{2}e^{2x}\right)+C=$

$=\frac{1}{2}x\,e^{2x}-\frac{1}{4}e^{2x}+C$

4.3 $\int e^x \cos x dx$

Seja $u = \cos x \Rightarrow du = -\sin x dx$

$dv = e^x \, dv \Rightarrow v = e^x$

$\int e^x \cos x dx = e^x \cos x - \int\left(-e^x \sin x\right) dx =$

$= e^x \cos x + \int e^x \sin x dx$

Seja $u = \sin x \Rightarrow du = \cos x dx$

$dv = e^x \, dv \Rightarrow v = e^x$

$\int e^x \cos x dx = e^x \cos x + \left[e^x \sin x - \left(e^x \cos x\right) dx\right] =$

$= e^x \cos x + e^x \sin x - \int e^x \cos x dx$

Considerando $I = \int e^x \cos x dx$, vem:

$I = e^x \cos x + e^x \sin x - I$ ou seja:

$2I = e^x \cos x + e^x \sin x$

Logo,

$I = \frac{1}{2}\left(e^x \cos x + e^x \sin x\right) + C$

$\int e^x \cos x \, dx = \frac{1}{2} e^x \left(\cos x + \sin x\right) + C$

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1. $\int 3x^5 dx = 3\int x^5 dx =$

$= 3\frac{x^{5+1}}{5+1} + C = 3\frac{x^6}{6} + C = \frac{1}{2}x^6 + C$

Resposta: (A)

2. $\int\left(3x^3 - 5x^2 + 3\right) dx =$

$= 3\int x^3 dx - 5\int x^2 dx + 3\int dx =$

$= 3\frac{x^4}{4} - 5\frac{x^3}{3} + 3x + C$

$= \frac{3}{4}x^4 - \frac{5}{3}x^3 + 3x + C$

Resposta: (C)

3. $\int \frac{\sqrt{2x}}{\sqrt[3]{5x}} dx = \frac{\sqrt{2}}{\sqrt[3]{5}} \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx =$

$= \frac{\sqrt{2}}{\sqrt[3]{5}} \int x^{\frac{1}{6}} dx = \frac{\sqrt{2}}{\sqrt[3]{5}} \times \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1} + C =$

$= \frac{\sqrt{2}}{\sqrt[3]{5}} \times \frac{6}{7} x^{\frac{7}{6}} + C = \frac{6\sqrt{2}\sqrt[6]{x^7}}{7\sqrt[3]{5}} + C$

Resposta: (B)

4. $\int\left(2^x - \frac{1}{x}\right) dx = \int 2^x dx - \int \frac{1}{x} dx = \frac{2^x}{\ln 2} - \ln|x| + C$

Resposta: (C)

5. $\int \sqrt{e^x} dx = \int e^{\frac{x}{2}} dx = 2e^{\frac{x}{2}} + C = 2\sqrt{e^x} + C$

Resposta: (B)

6. $\int x^2 e^{2x} dx$

Seja $u = x^2 \Rightarrow du = 2x dx$

$dv = e^{2x} dx \Rightarrow v = \frac{1}{2}e^{2x}$

$\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int\left(\frac{1}{2}e^{2x} \times 2x\right) dx =$

$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x} dx$

Seja $u = x \Rightarrow du = dx$

$dv = e^{2x} dx \Rightarrow v = \frac{1}{2}e^{2x} dx$

$\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \left[\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx\right] =$

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\int e^{2x} dx =$

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C =$

$= e^{2x}\left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right) + C =$

Resposta: (D)

7. $\int x^2 \sin x dx$

Seja $u = x^2 \Rightarrow du = 2x dx$

$dv = \sin x dx \Rightarrow v = -\cos x$

$\int x^2 \sin x dx = -x^2 \cos x - \int\left(-2x \cos x\right) dx =$

$= -x^2 \cos x + 2\int x \cos x dx$

Seja $u = x \Rightarrow du = dx$

$dv = \cos x dx \Rightarrow v = \sin x$

$\int x^2 \sin x dx = -x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right] =$

$= -x^2 \cos x + 2\left(x \sin x + \cos x\right) + C =$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C$

Resposta: (A)

8. $\int \frac{\ln x}{x^2} dx$

Seja $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = \frac{1}{x^2} dx \Rightarrow dv = x^{-2} dx \Rightarrow v = -x^{-1}$

$\int \frac{\ln x}{x^2} dx = -x^{-1} \ln x - \int \left(-x^{-1} \times \frac{1}{x} \right) dx =$

$= -\frac{\ln x}{x} + \int x^{-2} dx =$

$= -\frac{\ln x}{x} - x^{-1} + C =$

$= -\frac{\ln x}{x} - \frac{1}{x} + C$

Resposta: (B)

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1.1 $\int x^2 dx = \frac{x^{2+1}}{2+1} + C = \frac{1}{3} x^3 + C$

1.2 $\int x^7 dx = \frac{x^{7+1}}{7+1} + C = \frac{1}{8} x^8 + C$

1.3 $\int x dx = \frac{x^{1+1}}{1+1} + C = \frac{1}{2} x^2 + C$

1.4 $\int \sqrt{x} dx = x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3} \sqrt{x^3} + C$

1.5 $\int (3x^5) dx = 3 \int x^5 dx = 3 \frac{x^{5+1}}{5+1} + C = \frac{1}{2} x^6 + C$

1.6 $\int \frac{7}{x^3} dx = 7 \int x^{-3} dx = 7 \frac{x^{-3+1}}{-3+1} + C =$

$= -\frac{7}{2} x^{-2} + C = -\frac{7}{2x^2} + C$

1.7 $\int \sqrt{x^3} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C =$

$= \frac{2}{5} x^{\frac{5}{2}} + C = \frac{2}{5} \sqrt{x^5} + C$

1.8 $\int \frac{1}{\sqrt[3]{x^2}} dx = \int x^{-\frac{2}{3}} dx = \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C =$

$= 3x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$

1.9 $\int \frac{5x^5 - 2x^4 + 3x^2 + 4}{x^2} dx =$

$= \int \left(5x^3 - 2x^2 + 3 + 4x^{-2} \right) dx =$

$= 5 \int x^3 dx - 2 \int x^2 dx + 3 \int dx + 4 \int x^{-2} dx =$

$= \frac{5}{4} x^4 - \frac{2}{3} x^3 + 3x + \frac{4}{x} + C$

1.10 $\int \sin x dx = -\cos x + C$

1.11 $\int \cos x dx = \sin x + C$

1.12 $\int \frac{1+x}{x} dx = \int \left(\frac{1}{x} + 1 \right) dx = \int \frac{1}{x} dx + \int dx = \ln|x| + x + C$

1.13 $\int e^x dx = e^x + C$

1.14 $\int a^x dx = \frac{a^x}{\ln a} + C$

1.15 $\int 2^x dx = \frac{2^x}{\ln 2} + C$

1.16 $\int \left(x^2 + \frac{5}{\sqrt{x}} \right) dx = \int x^2 dx + \int \left(5x^{-\frac{1}{2}} \right) dx$

$= \frac{x^3}{3} + 5 \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^3}{3} + 5 \times 2\sqrt{x} + C =$

$= \frac{x^3}{3} + 10\sqrt{x} + C$

1.17 $\int \left(\frac{1+\sqrt{x}}{x} \right)^2 dx = \int \frac{\left(1+\sqrt{x}\right)^2}{x^2} dx = \int \frac{1+x+2\sqrt{x}}{x^2} dx =$

$= \int \left(\frac{1}{x^2} + \frac{x}{x^2} + \frac{2\sqrt{x}}{x^2} \right) dx =$

$= \int \left(x^{-2} + x^{-1} + 2x^{\frac{1}{2}-2} \right) dx =$

$= \int x^{-2} dx + \int \frac{1}{x} dx + 2 \int x^{-\frac{3}{2}} dx =$

$= \frac{x^{-2+1}}{-1} + \ln|x| + 2 \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$

$= -\frac{1}{x} - 4x^{-\frac{1}{2}} + \ln|x| + C =$

$= -\frac{1}{x} - \frac{4}{\sqrt{x}} + \ln|x| + C$

1.18 $\int\left(1+\frac{1}{x^3}+\frac{2}{x\sqrt{x}}\right)dx=\int 1dx+\int\frac{1}{x^3}dx+\int\frac{2}{x\sqrt{x}}dx=$

$=\int 1dx+\int x^{-3}dx+2\int x^{-\frac{3}{2}}dx=$

$=x+\frac{x^{-2}}{-2}+2\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}+C$

$=x-\frac{1}{2x^2}-\frac{4}{\sqrt{x}}+C$

1.19 $\int\frac{\left(\sqrt{2}-\sqrt{x}\right)^2}{\sqrt{2x}}dx=\int\frac{2-2\sqrt{2}\sqrt{x}+x}{\sqrt{2}\sqrt{x}}dx=$

$=\int\frac{2}{\sqrt{2}\sqrt{x}}dx-\int\frac{2\sqrt{2}\sqrt{x}}{\sqrt{2}\sqrt{x}}dx+\int\frac{x}{\sqrt{2}\sqrt{x}}dx$

$=\frac{2}{\sqrt{2}}\int x^{-\frac{1}{2}}dx-2\int dx+\frac{1}{\sqrt{2}}\int x^{\frac{1}{2}}dx=$

$=\sqrt{2}\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}-2x+\frac{1}{\sqrt{2}}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=$

$=2\sqrt{2}\sqrt{x}-2x+\frac{\sqrt{2}}{3}x^{\frac{3}{2}}+C=$

$=2\sqrt{2}\sqrt{x}-2x+\frac{\sqrt{2}}{3}x\sqrt{x}+C=$

1.20 $\int e^{2x}\,dx=\frac{1}{2}\int 2e^{2x}\,dx=\frac{1}{2}\times e^{2x}+C=\frac{e^{2x}}{2}+C$

1.21 $\int\left(e^x+1\right)^2 dx=\int\left(e^{2x}+2e^x+1\right)dx=$

$=\int e^{2x}\,dx+\int 2e^x\,dx+\int 1dx=$

$=\frac{1}{2}\int 2e^{2x}\,dx+2\int e^x\,dx+\int 1dx=$

$=\frac{1}{2}e^{2x}+2e^x+x+C=$

$=\frac{e^{2x}}{2}+2e^x+x+C$

1.22 $\int 2^{3x}dx=\int 8^x dx=\frac{8^x}{\ln 8}+C=$

$=\frac{8^x}{\ln 2^3}+C=\frac{8^x}{3\ln 2}+C$

1.23 $\int\left(3^x+1\right)^2 dx=\int\left(3^{2x}+2\times 3^x+1\right)dx=$

$=\int 3^{2x}dx+\int 2\times 3^x dx+\int 1dx=$

$=\int 9^x dx+2\int 3^x dx+\int 1dx=$

$=\frac{9^x}{\ln 9}+2\frac{3^x}{\ln 3}+x+C=$

$=\frac{9^x}{2\ln 3}+\frac{2\times 3^x}{\ln 3}+x+C$

1.24 $\int\cos(2x)\,dx=\frac{1}{2}\int 2\cos(2x)\,dx=\frac{1}{2}\sin(2x)+C$

1.25 $\int\sin(2x)\,dx=-\frac{1}{2}\int(-2\sin(2x))\,dx=-\frac{1}{2}\cos(2x)+C$

1.26 $\int(1+\cos(2x-1))\,dx=\int dx+\frac{1}{2}\int 2\cos(2x-1)dx=$

$=x+\frac{1}{2}\sin(2x-1)+C$

1.27 $\int\left(\sqrt{x}+\sin(3x+2)\right)dx=$

$=\int x^{\frac{1}{2}}dx-\frac{1}{3}\int(-3\sin(3x+2))\,dx=$

$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{1}{3}\cos(3x+2)+C=$

$=\frac{2x\sqrt{x}}{3}-\frac{1}{3}\cos(3x+2)+C=$

1.28 $\int\frac{2+x\sin x}{x}dx=\int\frac{2}{x}dx+\int\frac{x\sin x}{x}dx=$

$=2\int\frac{1}{x}dx+\int\sin x dx=2\ln|x|-\cos x+C$

1.29 $\int\sin x\cos x\,dx=\frac{1}{2}\int 2\sin x\cos x\,dx=$

$=\frac{1}{2}\int\sin(2x)\,dx=\frac{1}{2}\times\left(-\frac{1}{2}\right)\int(-2\sin(2x))\,dx=$

$=-\frac{1}{4}\cos(2x)+C$

1.30 $\int(\sin x+\cos x)^2\,dx=$

$=\int\left(\sin^2 x+2\sin x\cos x+\cos^2 x\right)dx=$

$=\int\left(\sin^2 x+\cos^2 x+\sin(2x)\right)dx=$

$=\int(1+\sin(2x))\,dx=\int dx+\int\sin(2x)\,dx=$

$=\int dx-\frac{1}{2}\int(-2\sin(2x))\,dx=x-\frac{1}{2}\cos(2x)+C$

2.1 $\int x e^x \, dx$

Seja: $u = x \Rightarrow du = dx$

$dv = e^x \, dx \Rightarrow v = e^x$

$\int x e^x \, dx = x e^x - \int e^x \, dx =$

$= x e^x - e^x + C = e^x (x-1) + C$

2.2 $\int x (x+3)^3 \, dx$

Seja: $u = x \Rightarrow du = dx$

$dv = (x+3)^3 \, dx \Rightarrow v = \frac{1}{4}(x+3)^4$

$\int x(x+3)^3 \, dx = \frac{1}{4} x (x+3)^4 - \int \frac{1}{4}(x+3)^4 \, dx =$

$= \frac{1}{4} x (x+3)^4 - \frac{1}{4}\left(\frac{1}{5}(x+3)^5\right) + C =$

$= \frac{1}{4} x (x+3)^4 - \frac{1}{20}(x+3)^5 + C =$

$= \frac{1}{4}(x+3)^4 \left(x - \frac{1}{5}(x+3)\right) + C =$

$= \frac{1}{4}(x+3)^4 \left(\frac{4}{5}x - \frac{3}{5}\right) + C =$

$= \frac{1}{20}(x+3)^4 (4x-3) + C$

2.3 $\int x^3 \ln x dx$

Seja: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x^3 dx \Rightarrow v = \frac{1}{4} x^4$

$\int x^3 \ln dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \times \frac{1}{x} dx =$

$= \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^3 dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C =$

$= \frac{1}{16} x^4 (4 \ln x - 1) + C = \frac{x^4}{16}(4 \ln x - 1) + C$

2.4 $\int x^2 \ln x dx$

Seja: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x^2 dx \Rightarrow v = \frac{x^3}{3}$

$\int x^2 \ln dx = \frac{x^3}{3} \ln x - \int \left(\frac{x^3}{3} \times \frac{1}{x}\right) dx =$

$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$

2.5 $\int x \sin x dx$

Seja: $u = x \Rightarrow du = dx$

$dv = \sin x dx \Rightarrow v = -\cos x$

$\int x \sin x dx = -x \cos x - \int (-\cos x) \, dx =$

$= -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$

$= \sin x - x \cos x + C$

2.6 $\int x \cos x dx$

Seja: $u = x \Rightarrow du = dx$

$dv = \cos x dx \Rightarrow v = \sin x$

$\int x \cos x \, dx = x \sin x - \int \sin x \, dx =$

$= x \sin x + \cos x + C$

2.7 $\int \frac{\ln x}{x} dx$

Seja: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = \frac{1}{x} dx \Rightarrow v = \ln x$

$\int \frac{\ln x}{x} dx = \ln x \times \ln x - \int \frac{\ln x}{x} \, dx \Leftrightarrow$

$\Leftrightarrow \int \frac{\ln x}{x} dx = (\ln x)^2 - \int \frac{\ln x}{x} \, dx \Leftrightarrow$

$\Leftrightarrow 2 \int \frac{\ln x}{x} dx = (\ln x)^2 \Leftrightarrow \int \frac{\ln x}{x} dx = \frac{\ln^2 x}{2} + C$

2.8 $\int \frac{\ln x}{\sqrt{x}} dx$

Seja: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = \frac{1}{\sqrt{x}} dx = x^{-\frac{1}{2}} dx \Rightarrow v = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = 2x^{\frac{1}{2}}$

$\int \frac{\ln x}{\sqrt{x}} dx = 2x^{\frac{1}{2}} \ln x - \int \frac{2x^{\frac{1}{2}}}{x} dx =$

$= 2\sqrt{x} \ln x - 2 \int x^{-\frac{1}{2}} \, dx =$

$= 2\sqrt{x} \ln x - 2 \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C =$

$= 2\sqrt{x} \ln x - 4\sqrt{x} + C =$

$= 2\sqrt{x} (\ln x - 2) + C$

2.9 $\int \left(x^2 e^x\right) dx$

Seja: $u = x^2 \Rightarrow du = 2x dx$

$dv = e^x\, dx \Rightarrow v = e^x$

$\int \left(x^2 e^x\right) dx = x^2 e^x - \int \left(2x e^x\right) dx = x^2 e^x - 2\int \left(x e^x\right) dx$

Seja: $u = x \Rightarrow du = dx$

$dv = e^x\, dx \Rightarrow v = e^x$

$x^2 e^x - 2\int \left(x e^x\right) dx = x^2 e^x - 2\left[x e^x - \int e^x\, dx\right] =$

$= x^2 e^x - 2x e^x + 2 e^x + C = e^x \left(x^2 - 2x + 2\right) + C$

2.10 $\int \frac{x}{3^x} dx = \int x \times 3^{-x}\, dx$

Seja: $u = x \Rightarrow du = dx$

$dv = 3^{-x} dx \Rightarrow v = \frac{-3^x}{\ln 3}$

$\int \frac{x}{3^x} dx = x \frac{-3^x}{\ln 3} - \int \frac{-3^x}{\ln 3} dx =$

$= \frac{-3^x x}{\ln 3} + \frac{1}{\ln 3} \int 3^x\, dx =$

$= \frac{-3^x x}{\ln 3} + \frac{1}{\ln 3} \frac{-3^x}{\ln 3} + C =$

$= 3^x \left(\frac{-x}{\ln 3} - \frac{1}{\ln^2 3}\right) + C$

2.11 $\int (\ln x)^2\, dx$

Seja: $u = (\ln x)^2 \Rightarrow du = 2(\ln x)\frac{1}{x} dx$

$dv = dx \Rightarrow v = x$

$\int (\ln x)^2\, dx = (\ln x)^2\, x - \int 2x(\ln x)\frac{1}{x} dx =$

$x \ln^2 x - 2\int \ln x dx$

Seja: $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = dx \Rightarrow v = x$

$x \ln^2 x - 2\int \ln x\, dx = x \ln x - \int x \times \frac{1}{x}\, dx =$

$= x \ln x - \int dx = x \ln x - x + C$

Logo, $\int (\ln x)^2\, dx = x \ln x - x + C$

2.12 $\int e^{-x} \cos x\, dx = I$

Seja $u = \cos x \Rightarrow du = -\sin x\, dx$

$dv = e^{-x}\, dx \Rightarrow v = -e^{-x}$

$I = -e^{-x} \cos x - \int \left(-e^{-x}\right)\left(-\sin x\right) dx =$

$= -e^{-x} \cos x - \int e^{-x} \sin x\, dx$

Seja: $u = \sin x \Rightarrow du = \cos x\, dx$

$dv = e^{-x}\, dx \Rightarrow v = -e^{-x}$

$I = -e^{-x} \cos x - \left[-e^{-x} \sin x - \int \left(-e^{-x} \cos x\right) dx\right] \Leftrightarrow$

$\Leftrightarrow I = -e^{-x} \cos x - \left[-e^{-x} \sin x - \int \left(-e^{-x} \cos x\right) dx\right] \Leftrightarrow$

$\Leftrightarrow I = -e^{-x} \cos x + e^{-x} \sin x - \int e^{-x} \cos x\, dx \Leftrightarrow$

$\Leftrightarrow I = -e^{-x} \cos x + e^{-x} \sin x - I + C \Leftrightarrow$

$\Leftrightarrow 2I = e^{-x}\left(-\cos x + \sin x\right) + C \Leftrightarrow$

$\Leftrightarrow I = \frac{e^{-x}}{2}\left(\sin x - \cos x\right) + C$