PROPOSTAS DE RESOLUÇÃO

Capítulo 1

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1.1 $\left|-2-(-3)\right|^2 = \left|-2+3\right|^2 = 1^2 = 1$

1.2 $\left|-1\right|^{\frac{1}{2}} + \left|-(-1)\right|^3 = 1^{\frac{1}{2}} + 1^3 = \sqrt{1}+1 = 2$

1.3 $\left|-1-3\right|^2 - \left|-(-4)\right|^{\frac{1}{2}} = \left|-4\right|^2 - 4^{\frac{1}{2}} = 4^2 - \sqrt{4} = 14$

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2.1 $|x-1| = 3 \Leftrightarrow x-1 = 3 \vee x-1 = -3$

$\Leftrightarrow x = 4 \vee x = -2$

$S = \{-2, 4\}$

2.2 $\left|-\frac{1}{2}x-1\right| = \frac{1}{3} \Leftrightarrow -\frac{1}{2}x-1 = \frac{1}{3} \vee -\frac{1}{2}x-1 = -\frac{1}{3}$

$\Leftrightarrow -\frac{1}{2}x = \frac{4}{3} \vee -\frac{1}{2}x = \frac{2}{3}$

$\Leftrightarrow x = -\frac{8}{3} \vee x = -\frac{4}{3}$

$S = \left\{-\frac{8}{3}, -\frac{4}{3}\right\}$

2.3 $\left|-2x+\frac{1}{2}x^2\right| = 0 \Leftrightarrow -2x+\frac{1}{2}x^2 = 0 \Leftrightarrow x\left(\frac{1}{2}x-2\right) = 0$

$\Leftrightarrow x = 0 \vee \frac{1}{2}x-2 = 0 \Leftrightarrow x = 0 \vee x = 4$

$S = \{0, 4\}$

2.4 $\left|-1-2x\right| = -1 \Leftrightarrow x \in \varnothing$

$S = \{\}$

2.5 $\left|2x-1\right| > 0 \Leftrightarrow 2x-1 \neq 0 \Leftrightarrow x \neq \frac{1}{2}$

$S = \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$

2.6 $\left|2x-1\right| < 3 \Leftrightarrow 2x-1 < 3 \wedge 2x-1 > -3$

$\Leftrightarrow 2x < 4 \wedge 2x > -2 \Leftrightarrow x < 2 \wedge x > -1$

$\Leftrightarrow x \in \left]-1, 2\right[$

$S = \left]-1, 2\right[$

2.7 $\left|-2x+\frac{1}{2}\right| > 5 \Leftrightarrow -2x+\frac{1}{2} > 5 \vee -2x+\frac{1}{2} < -5$

$\Leftrightarrow -4x+1 > 10 \vee -4x+1 < -10$

$\Leftrightarrow -4x > 9 \vee -4x < -11$

$\Leftrightarrow x < -\frac{9}{4} \vee x > \frac{11}{4}$

$S = \left]-\infty, -\frac{9}{4}\right[\cup \left]\frac{11}{4}, +\infty\right[$

2.8 $\left|-x+\frac{1}{2}\right| > 7 \Leftrightarrow \left|-x+\frac{1}{2}\right| > 7$

$\Leftrightarrow -x+\frac{1}{2} > 7 \vee -x+\frac{1}{2} < -7$

$\Leftrightarrow -x > \frac{13}{2} \vee -x < -\frac{15}{2}$

$\Leftrightarrow x < -\frac{13}{2} \vee x > \frac{15}{2}$

$S = \left]-\infty, -\frac{13}{2}\right[\cup \left]\frac{15}{2}, +\infty\right[$

2.9 $\left|-2x+3\right| < -5 \Leftrightarrow x \in \varnothing$

$S = \{\}$

2.10 $\left|-3x+1\right| > -8 \Leftrightarrow x \in \mathbb{R}$

$S = \mathbb{R}$

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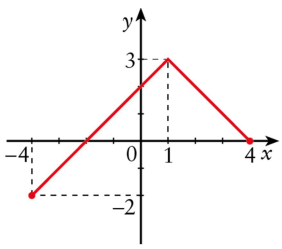
3.1 **a)** $f(x) = \begin{cases} -(x-1)+3 & \text{se } x-1 \geq 0 \wedge x \in [-4, 4] \\ -[-(x-1)]+3 & \text{se } x-1 < 0 \wedge x \in [-4, 4] \end{cases}$

$f(x) = \begin{cases} -x+4 & \text{se } x \in [1, 4] \\ x+2 & \text{se } x \in [-4, 1[\end{cases}$

b)

x	$y = -x + 4$
1	3
4	0

x	$y = x + 2$
– 4	– 2
1	3



c) $D'_f = [-2, 3]$

d) Zeros: – 2 e 4.

Monotonia: f é estritamente crescente em $[-4, 1]$ e é estritamente decrescente em $[1, 4]$.

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3.2 **a)** $\left|x-1\right|-1 = \begin{cases} x-1-1 & \text{se } x-1 \geq 0 \\ -x+1-1 & \text{se } x-1 < 0 \end{cases}$

$\Leftrightarrow \left|x-1\right|-1 = \begin{cases} x-2 & \text{se } x \geq 1 \\ -x & \text{se } x < 1 \end{cases}$

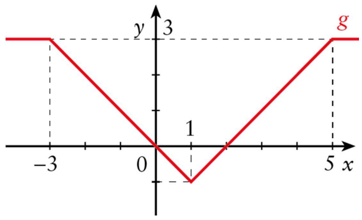
Logo,

$g(x) = \begin{cases} -x & \text{se } -3 \leq x < 1 \\ x-2 & \text{se } 1 \leq x \leq 5 \\ 3 & \text{se } x < -3 \vee x > 5 \end{cases}$

Gráficos:

x	$y = -x$
– 3	3
1	– 1

x	$y = x - 2$
1	– 1
5	3



b) Zeros: 0 e 2.

Monotonia: g é estritamente decrescente em [– 3, 1], estritamente crescente em [1, 5] e constante em $]-\infty, -3]$ e em $[5, +\infty[$.

4.1 $f(x) = -|x-3| + 5$

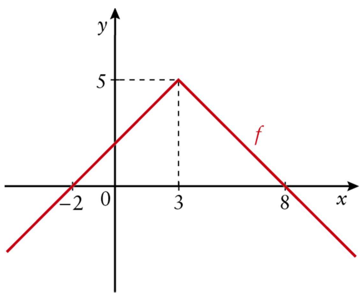
$$f(x) = \begin{cases} -x+8 & \text{se } x \geq 3 \\ x+2 & \text{se } x < 3 \end{cases}$$

$y = -x + 8$

x	y
3	5
8	0

$y = x + 2$

x	y
3	5
– 2	0



4.2. $g(x) = -\left|\frac{1}{2}x\right| - 3$

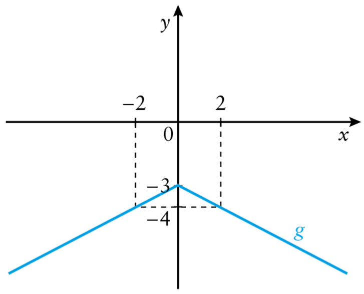
$$g(x) = \begin{cases} -\frac{1}{2}x - 3 & \text{se } x \geq 0 \\ \frac{1}{2}x - 3 & \text{se } x < 0 \end{cases}$$

$y = -\frac{1}{2}x - 3$

x	y
0	– 3
2	– 4

$y = \frac{1}{2}x - 3$

x	y
0	– 3
2	– 2



4.3. $h(x) = -2|x| + 3$

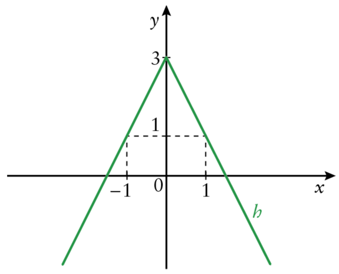
$$h(x) = \begin{cases} -2x+3 & \text{se } x \geq 0 \\ 2x+3 & \text{se } x < 0 \end{cases}$$

$y = -2x + 3$

x	y
0	3
1	1

$y = 2x + 3$

x	y
0	3
– 1	1



4.4 $i(x) = -|2x-3| + 1$

$$i(x) = \begin{cases} -(2x-3)+1 & \text{se } x \geq \frac{3}{2} \\ -(-2x+3)+1 & \text{se } x < \frac{3}{2} \end{cases}$$

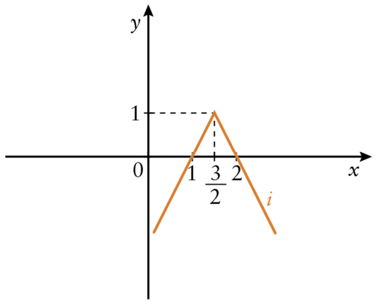
$$i(x) = \begin{cases} -2x+4 & \text{se } x \geq \frac{3}{2} \\ 2x-2 & \text{se } x < \frac{3}{2} \end{cases}$$

$y = -2x + 4$

x	y
$\frac{3}{2}$	1
2	0

$y = 2x - 2$

x	y
$\frac{3}{2}$	1
1	0



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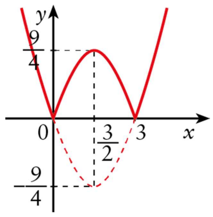
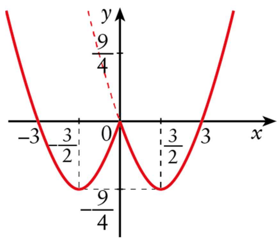
5.1 **a)**



b)



5.2 Pode. Seja, por exemplo, a função definida, em $\mathbb{R}$, por $f(x) = x^4 - x^2$.

f tem três zeros: – 1, 0 e 1 e é par,

$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$, qualquer que seja o valor real de x.

6.1 $|x-3| = |x-1| \Leftrightarrow x-3 = x-1 \vee x-3 = -(x-1) \Leftrightarrow$

$\Leftrightarrow 0x = 2 \vee x-3 = -x+1 \Leftrightarrow$

$\Leftrightarrow 2x = 4 \Leftrightarrow x = 2$

$S = \{2\}$

6.2 $2|x-3| = -3|2x-1| \Leftrightarrow$

$|2x-6| = -|6x-3| \Leftrightarrow$

$|2x-6| + |6x-3| = 0$

A soma de dois números não negativos, $|2x-6|$ e $|6x-3|$, só é nulo se os dois números o forem simultaneamente, ou seja:

$|2x-6| = 0 \wedge |6x-3| = 0 \Leftrightarrow 2x-6 = 0 \wedge 6x-3 = 0 \Leftrightarrow$

$\Leftrightarrow x = 3 \wedge x = \frac{1}{2}$, impossível.

$S = \{\}$

7.1 $2|x-3| \geq 3|x+1| \Leftrightarrow |2x-6| - |3x+3| \geq 0$

Seja $f(x) = |2x-6| - |3x+3| \Leftrightarrow$

$$\Leftrightarrow f(x) = \begin{cases} x+9 & \text{se } x \leq -1 \\ -5x+3 & \text{se } -1 < x < 3 \\ -x-9 & \text{se } x \geq 3 \end{cases}$$

$y = x + 9$

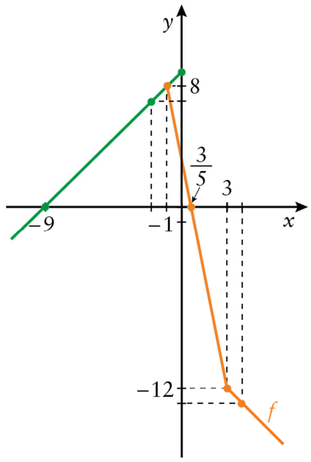
x	y
– 1	8
– 2	7

$y = -5x + 3$

x	y
– 1	8
3	– 12

$y = -x - 9$

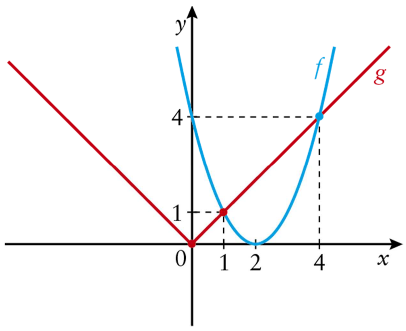
x	y
3	– 12
4	– 13



$f(x) \geq 0 \Leftrightarrow x \in \left[-9, \frac{3}{5}\right]$

7.2 $|x-2|^2 - |x| \leq 0 \Leftrightarrow (x-2)^2 \leq |x|$

Seja $f(x) = (x-2)^2$ e $g(x) = |x|$



$|x-2|^2 - |x| \leq 0 \Leftrightarrow |x-2|^2 \leq |x| \Leftrightarrow f(x) \leq g(x) \Leftrightarrow$

$x \in [1, 4]$

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1. $f(x) = |1-x| - 4$; $\quad g(x) = |f(x)|$

$g(2) = |f(2)| = ||1-2| - 4| = |1-4| = 3$

Resposta: (D)

2. $D'_f = [-1, 2]$

Resposta: (B)

3. $f(x)=g(x)\Leftrightarrow$

$\Leftrightarrow |x-1|=|2x-3|\Leftrightarrow$

$\Leftrightarrow x-1=2x-3 \vee x-1=-2x+3\Leftrightarrow$

$\Leftrightarrow -x=-2 \vee 3x=4\Leftrightarrow$

$\Leftrightarrow x=2 \vee x=\frac{4}{3}$

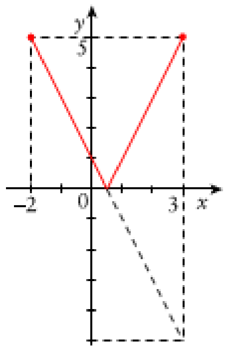
Resposta: (D)

4. $f(x) = -2x + 1$

$D'_f = [-2, 3]$

$f(-2) = -2 \times (-2) + 1 = 5$

$f(3) = -2 \times 3 + 1 = -5$



Resposta: (C)

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1. $A(-3, 4)$; $B(0, 1)$ e $C(2, 3)$.

$m=\frac{4-1}{-3-0}=-1$

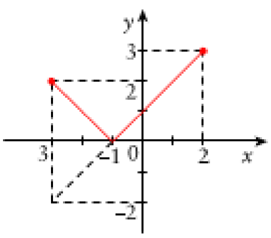
$y-1=-1(x-0)\Leftrightarrow y=-x+1$

$m=\frac{1-3}{0-2}=1$

$y-1=-1(x-0)\Leftrightarrow y=x+1$

2.1 $f(x)=\begin{cases}2 & \text{se } -9\le x\le -3\\ -x+1 & \text{se } -3<x\le 0\\ x+1 & \text{se } 0<x\le 2\\ 3 & \text{se } 2<x\le 9\end{cases}$

x	$x + 1$
-3	-2
2	3

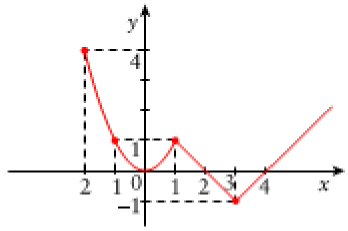


2.2

$g(x)=\begin{cases}|x-3|-1 & \text{se } x>1\\ x^2 & \text{se } -2\le x\le 1\end{cases}$

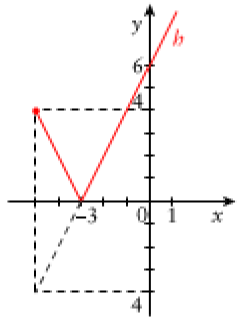
$g(x)=\begin{cases}x-3-1 & \text{se } x\ge 3\wedge x>1\\ -x+3-1 & \text{se } x<3\wedge x>1\\ x^2 & \text{se } -2\le x\le 1\end{cases}$

$g(x)=\begin{cases}x-4 & \text{se } x\ge 3\\ -x+2 & \text{se } 1<x<3\\ x^2 & \text{se } -2\le x\le 1\end{cases}$



3.1 $h(x)=2|x+3|\,,\ x\in[-5,+\infty[$

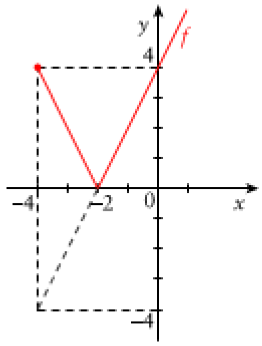
x	$y = 2x + 6$
-5	-4
-3	0
0	6



3.2 a) $h(x-1)=2|x-1+3|\wedge x-1\ge -5$

$f(x)=2|x+2|\wedge x\ge -4$

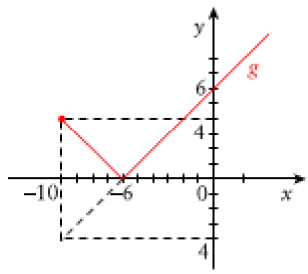
x	$y = 2x + 4$
-4	-4
-2	0
0	4



b) $h\left(\frac{x}{2}\right)=2\left|\frac{x}{2}+3\right|\wedge \frac{x}{2}\ge -5$

$g(x)=|x+6|\wedge x\ge -10$

x	$y = x + 6$
-10	-4
-6	0
0	6



4. $D_h = [-5, +\infty[$; $D'_h = [0, +\infty[$; Zeros: $\{-3\}$; h é decrescente em [– 5, – 3] e crescente em $[-3, +\infty[$.

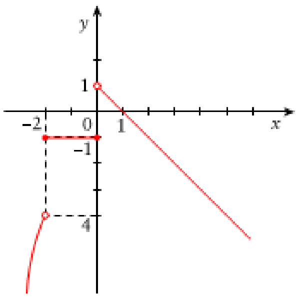
$D_f = [-4, +\infty[$; $D'_f = [0, +\infty[$; Zeros: $\{-2\}$; f é decrescente em [– 4, – 2] e crescente em $[-2, +\infty[$.

$D_g = [-10, +\infty[$; $D'_g = [0, +\infty[$; Zeros: $\{-6\}$; g é decrescente em [– 10, – 6] e crescente em $[-6, +\infty[$.

5.

$$f(x) = \begin{cases} -x+1 & \text{se } x > 0 \\ -1 & \text{se } -2 \le x \le 0 \\ -x^2 & \text{se } x < -2 \end{cases}$$

5.1



5.2 $D'_f =]-\infty, 1[$

5.3 $f(x) = 0 \Leftrightarrow x = 1$

Resposta: {1}

5.4 $D'_g = [0, +\infty[$

5.5 $h(x) = -|3x| = \begin{cases} -3x & \text{se } x > 0 \\ 3x & \text{se } x \le 0 \end{cases}$

Para $x < -2$,

$f(x) = h(x) \Leftrightarrow -x^2 = 3x \wedge x < -2$

$\Leftrightarrow 3x + x^2 = 0 \wedge x < -2$

$\Leftrightarrow x(3 + x) = 0 \wedge x < -2$

$\Leftrightarrow x = -3$

$f(-3) = -9$

Para $-2 \le x \le 0$,

$f(x) = h(x) \Leftrightarrow 3x = -1 \wedge -2 \le x \le 0 \Leftrightarrow x = -\frac{1}{3}$

$f\left(-\frac{1}{3}\right) = -1$

Para $x > 0$,

$f(x) = h(x) \Leftrightarrow -x + 1 = -3x \wedge x > 0$

$\Leftrightarrow 2x = -1 \wedge x > 0$

$\Leftrightarrow x \in \varnothing$

Resposta: (– 3, – 9) e $\left(-\frac{1}{3}, -1\right)$.

6. 100 km : 100 × 1 Mt = 100 Mt

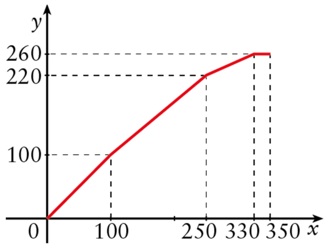
250 km : 100 Mt + 150 × 0,8 Mt = 220 Mt

330 km : 220 Mt + 80 × 0,5 Mt = 260 Mt

350 km : 260 Mt

6.1

$$f(x) = \begin{cases} x & \text{se } 0 \le x \le 100 \\ 100 + 0{,}8(x-100) & \text{se } 100 < x \le 250 \\ 220 + 0{,}5(x-250) & \text{se } 250 < x \le 330 \\ 260 + 0{,}5(x-330) & \text{se } 330 < x \le 400 \\ 274 & \text{se } 400 < x \le 430 \end{cases}$$



6.2 f (300) = 200 + 0,5 (300 – 250) = 245 Mt

3 × 245 Mt = 735 Mt

Resposta: 735 Mt

6.3 f (175) = 100 + 0,8 × (175 – 100) = 100 + 0,8 × 75

= 160 Mt ⟵ custo de uma tonelada.

Logo, $C(x) = 160x$ (meticais).